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HEAT-TRANSFER INTENSITY FROM SWIRLING DISPERSE  
FLOW TO CYCLONE-CHAMBER WALL

V. A. Kirakosyan, A. P. Baskakov,  
E. Yu. Lavrovskaya, and Yu. A. Popov

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On the basis of a two-layer scheme of turbulent motion around a wall, an expression is obtained for the heat-transfer coefficient from swirling disperse flow to the cyclone wall.

In engineering practice, the heat extraction from the lateral surface of the cyclone must often be estimated in calculating designing a cyclone-type heat exchanger. Accurate calculation of the heat fluxes from swirling disperse flow to the cyclone wall requires combined solution of the equations of energy and motion of the gas and the particles. Analytical solution of this problem is not possible. Usually, various empirical dependences which are only valid for the values of the cyclone structural parameters corresponding to the particular experiment [1-4] are used to calculate the convective heat-transfer coefficient from the gases to the lateral surface of the cyclone. In addition the empirical dependences proposed in the literature take no account of the influence of dust content on the heat-transfer coefficient. It is expedient to generalize these experimental data using the well-known methods of approximate solution of analogous problems - in particular, the two-layer scheme of turbulent motion around a wall [5]. Following this method, it is first assumed that, on reaching the cyclone, the gas and particles move downward to the wall region of the apparatus (as indicated by experiments) and are of identical temperature.

By analogy with the problem of flow around a plane plate, it is assumed that the heat flux and tangential stress remain constant over the boundary layer and are equal to the corresponding values at the cyclone surface ( $q = q_{wa}$ ,  $\tau = \tau_{wa}$ ). Then the following expressions may be written

$$\frac{\tau_{wa}}{\rho} = -(v + v_r) \frac{1}{r} \frac{\partial(Vr)}{\partial r}, \quad (1)$$

$$\frac{q_{wa}}{\rho} = -C_{ef} (a + a_r) \frac{\partial T}{\partial r}, \quad (2)$$

where  $C_{ef} = C_g(1 + \mu C_s/C_g)$  is the effective specific heat of the disperse flow (it is assumed that, on account of the high intensity of interphase heat transfer, the temperature of the gas and the particles is the same).

Introducing the notation

$$V_* = \frac{\sqrt{\bar{v}wa}}{\rho}, \quad T_* = \frac{\rho wa}{\rho V_* C_{ef}}; \quad \eta = \frac{r}{R_{cy}}; \quad Re_* = \frac{V_* 2R_{cy}}{v},$$

$$Pr = \frac{v}{a}; \quad Pr_T = \frac{v_T}{a_T}; \quad \bar{V} = \frac{V}{V_*}; \quad \bar{T} = \frac{T}{T_*},$$

Eqs. (1) and (2) take the form

$$\left(1 + \frac{v_T}{v}\right) \frac{2}{Re_*} \frac{1}{\eta} \frac{\partial(\bar{V}\eta)}{\partial\eta} = -1, \quad (3)$$

$$\frac{2}{Pr Re_*} \left(1 + \frac{v_T}{v} \frac{Pr}{Pr_T}\right) \frac{\partial\bar{T}}{\partial\eta} = -1. \quad (4)$$

On the basis of the two-layer scheme of the wall zone of the cyclone chamber, a boundary layer of thickness  $1 - \eta_b$  ( $\eta_b = r_b/R_{cy}$  is the dimensionless boundary of the boundary layer) may be divided into a laminar sublayer with boundary  $\eta_0$  and a turbulent boundary layer. Since  $v_T/v \ll 1$  may be assumed in the laminar sublayer at the wall and  $v/v_T \ll 1$  in the turbulent boundary layer, Eqs. (3) and (4) take the form

$$\text{when } \eta_0 \leq \eta \leq 1 \quad \begin{cases} \frac{\partial(\bar{V}\eta)}{\partial\eta} = -\frac{Re_*\eta}{2}, \\ \frac{\partial\bar{T}}{\partial\eta} = -\frac{Pr Re_*}{2}, \end{cases} \quad (5)$$

$$\text{when } \eta_b \leq \eta \leq \eta_0 \quad \begin{cases} \frac{\partial\bar{T}}{\partial\eta} = -\frac{v}{v_T} \frac{Re_* Pr_T}{2}, \\ \frac{1}{\eta} \frac{\partial(\bar{V}\eta)}{\partial\eta} = -\frac{v}{v_T} \frac{Re_*}{2}. \end{cases} \quad (6)$$

Integration of Eqs. (5) and (6), for constant  $Pr$  and  $Pr_T$  under the condition that

$$\text{when } \eta = 1 \quad \bar{V} = 0, \quad \bar{T} = \bar{T}_{wa},$$

$$\text{when } \eta = \eta_b, \quad \bar{V} = \bar{V}_b, \quad \bar{T} = \bar{T}_b,$$

gives

$$\text{when } \eta_0 \leq \eta \leq 1 \quad \begin{cases} \bar{V} = \frac{Re_*}{2} \frac{1 - \eta^2}{2\eta}, \\ \bar{T} = \bar{T}_{wa} + Pr Re_* \frac{(1 - \eta)}{2}, \end{cases} \quad (7)$$

$$\text{when } \eta_b \leq \eta \leq \eta_0 \quad \begin{cases} \bar{V}\eta - \bar{V}_b\eta_b = -\frac{Re_*}{2} \frac{v}{v_T} \frac{\eta^2 - \eta_b^2}{2}, \\ \bar{T} - \bar{T}_b = -\frac{v}{v_T} \frac{Re_*}{2} Pr_T (\eta - \eta_b). \end{cases} \quad (8)$$

Denoting  $\bar{V}$  and  $\bar{T}$  at the laminar-sublayer boundary  $\eta_0$  by  $\bar{V}_0$  and  $\bar{T}_0$ , respectively, it follows from Eq. (8) that

$$\frac{\bar{V}_b\eta_b - \bar{V}_0\eta_0}{\bar{T}_b - \bar{T}_0} = \frac{\eta_0 + \eta_b}{2Pr_b}. \quad (9)$$

Substituting  $\bar{V}_0$  and  $\bar{T}_0$  from Eq. (7) into Eq. (9) gives

$$\frac{\bar{V}_b\eta_b - \frac{1}{2} Re_* \frac{1 - \eta_0^2}{2}}{\bar{T}_b - \bar{T}_{wa} - \frac{1}{2} Pr Re_* (1 - \eta_0)} = \frac{\eta_0 + \eta_b}{2 Pr_T},$$

or

$$\bar{T}_b - \bar{T}_{wa} = \frac{1}{2} Pr Re_* (1 - \eta_0) + \frac{2 Pr_T}{\eta_0 + \eta_b} \left[ \bar{V}_b\eta_b - \frac{Re_* (1 - \eta_0^2)}{4} \right]. \quad (10)$$

Since

$$\bar{T}_b = \frac{T_b}{T_*} = \frac{T_b \rho C_{ef} V_*}{q_{wa}}; \quad \bar{T}_{wa} = \frac{T_{wa}}{T_*} = \frac{\bar{T}_{wa} \rho V_* C_{ef}}{q_{wa}},$$

it follows from Eq. (10) that

$$T_b - T_{wa} = \frac{q_{wa}}{\rho C_{ef} V_*} \left\{ \text{Pr} \frac{\text{Re}_*}{2} (1 - \eta_0) + \frac{2 \text{Pr}_\tau}{\eta_0 + \eta_b} \left[ \bar{V}_b \eta_b - \frac{\text{Re}_* (1 - \eta_0^2)}{4} \right] \right\}. \quad (11)$$

Taking into account that  $\alpha = q_{wa}/(T_b - T_{wa})$  and  $\text{Nu} = \alpha 2R_{cy}/\lambda$ , the local value of the heat-transfer coefficient from the gas to the wall may be determined from the formula

$$\text{Nu} = \frac{\text{Re}_* \text{Pr} \left( 1 + \frac{C_s'}{C_g} \mu \right)}{\frac{\text{Pr} \text{Re}_*}{2} (1 - \eta_0) + \frac{2 \text{Pr}_\tau}{\eta_0 + \eta_b} \left[ \frac{V_b}{V_*} \eta_b - \frac{\text{Re}_* (1 - \eta_0^2)}{4} \right]}. \quad (12)$$

What is the expression for the laminar-sublayer thickness? As shown in [6], the tangential frictional stress in a swirling turbulent jet may be written in the form of a dependence on the variation in velocity circulation  $Vr$

$$\tau_{r\varphi} = \rho l^2 \left[ \frac{1}{r} \frac{\partial(Vr)}{\partial r} \right]^2.$$

Taking the mixing length in the cyclone boundary layer to be  $l = K(R_{cy} - r)$ , the following expression may be written

$$\text{when } \eta_b \leq \eta \leq \eta_0 \quad \frac{\tau_{wa}}{\rho V_{\max}^2} = \frac{K^2 (1 - \eta)^2}{\eta^2} \left( \frac{\partial \bar{V}}{\partial \eta} \right)^2, \quad (13)$$

and for the laminar sublayer, within which the law of laminar motion is valid, the corresponding expression is

$$\text{when } \eta_0 \leq \eta \leq 1 \quad \frac{\tau_{wa}}{\rho V_{\max}^2} = \frac{v}{R_{cy} V_{\max}} \left( \frac{\partial(V\eta)}{\partial \eta} \right). \quad (14)$$

Since the derivative of the velocity circulation undergoes a discontinuity on passing through the boundary of the laminar sublayer, it follows that

$$\left( \frac{\partial(V\eta)}{\partial \eta} \right)_{\eta=\eta_0+0} = K_1 \left( \frac{\partial(V\eta)}{\partial \eta} \right)_{\eta=\eta_0-0}. \quad (15)$$

Substituting Eqs. (13) and (14) into Eq. (15), it is found that

$$1 - \eta_0 = \frac{K_1}{K} \frac{v}{\sqrt{\frac{\tau_{wa}}{\rho} R_{cy}}}. \quad (16)$$

Let

$$C = \frac{1}{V_b} \sqrt{\frac{\tau_{wa}}{\rho}} = \frac{V_*}{V_b},$$

then  $\text{Re}_* = C(V_b/V_{in})\text{Re}$ , where  $\text{Re} = V_{in} 2R_{cy}/v$ .

Since the thickness of the boundary laminar sublayer is very small, i.e.,  $\eta_0 \approx 1$ , it follows that  $1 + \eta_0 \approx 2$ . Substituting Eq. (16) into Eq. (12) gives

$$\text{Nu} = \frac{C \frac{V_b}{V_{in}} \text{Re} \text{Pr} \left( 1 + \frac{C_s'}{C_g} \mu \right)}{\frac{K_1}{K} \text{Pr} + \frac{2 \text{Pr}_\tau}{1 + \eta_b - \frac{2K_1 V_{in}}{K V_b C \text{Re}}} \left[ \frac{\eta_b}{C} - \frac{K_1}{K} \right]}. \quad (17)$$

The frictional coefficient ( $f = 2C^2$ ) of a pure gas flow to the cyclone wall may be determined in the first approximation by analogy with flow around a plate. To take account of the influence of the dust content on the frictional coefficient in the tube, the intro-

duction of the factor  $(1 + \mu)^{0.56}$  was recommended in [6] on the basis of analysis of a large quantity of experimental material. However, in a cyclone, the particle concentration at the wall is considerably increased on account of separation. Taking this into account, the parameter C is approximated by the expression

$$C = \frac{1}{V_b} \sqrt{\frac{\tau_{wa}}{\rho}} = \left[ C_0 \frac{V_b(1 - \eta_b) R_{cy}}{v} \right]^{-m} (1 + A\mu)^{0.28},$$

where  $C_0$ ,  $m$ ,  $A$  are empirical coefficients.

Taking  $Pr_T = 1$ , it follows from Eq. (17) that

$$Nu = \frac{C_0 \left( \frac{V_b}{V_{in}} \right)^{1-m} \left( \frac{1 - \eta_b}{2} \right)^{-m} Pr Re^{1-m} \left( 1 + \frac{C_s}{C_g} \mu \right) (1 + A\mu)^{0.28}}{\frac{K_1}{K} Pr - \frac{2 \left[ \frac{K_1}{K} - \frac{\eta_b}{C_0} \left( \frac{V_b}{V_{in}} \right)^m \left( \frac{1 - \eta_b}{2} \right)^m Re^m \right]}{1 + \eta_b} - 2 \frac{K_1}{K} \frac{1}{C_0 \left( \frac{V_b}{V_{in}} \right)^{1-m} \left( \frac{1 - \eta_b}{2} \right)^{-m} Re^{1-m}}}, \quad (18)$$

and  $\eta_b$  and  $V_b/V_{in}$  are determined from the formulas proposed in [7]

$$\eta_b = 0,856 - 0,305h + 2,13h^2 - 8,22h^3, \quad (19)$$

where  $h = H/R_{cy}$  is the relative height of the input slits of the cyclone ( $0.004 \leq h \leq 0.24$ ), and

$$\frac{V_b}{V_{in}} = 0,769 \cdot 10^{2f} (\xi)^{-1,457} - 0,716 \cdot 10^{3f^2} (\xi)^{-2,914} - 0,398 \cdot 10^{5f^3} (\xi)^{-4,371} - 1,34(\eta_c - 0,5)^2, \quad (20)$$

where  $f = \Sigma f / (\pi R_{cy}^2)$  is the relative area of the input;  $\eta_c = R_c/R_{cy}$  is the relative radius of the contraction;  $\xi = L_{cy}/R_{cy}$  is the relative length of the cyclone ( $0.3 \leq \eta_c \leq 0.75$ ;  $0.4 \cdot 10^{-2} \leq f/\xi \leq 5.6 \cdot 10^{-2}$ ).

Equation (18) offers the possibility of determining the local value of the heat-transfer coefficient. If the boundary-layer thickness remains constant over the height of the cyclone, i.e.,  $\eta_b = \text{const}$ , then  $\alpha = \text{const}$ . Experimental determination of the boundary of the flow core [7] shows that this assumption is valid over practically the whole lateral surface of the cyclone chamber.

The coefficients  $C_0$ ,  $m$ ,  $K_1/K$  are determined by comparing the values of  $\alpha$  given by Eq. (18) with empirical dependences obtained in investigating cyclones of various sizes and

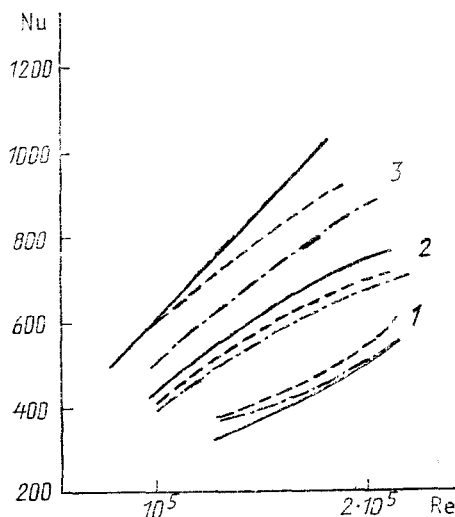


Fig. 1. Comparison of Eq. (18) (dashed curves) and Vyshenskii formula [1] (continuous curves) with the dependences of [2] (curve 1), [3] (curve 2), and [4] (curve 3).

TABLE 1. Comparison of Vyshenskii Empirical Dependence [1] and Eq. (18)

$f$	$Re \cdot 10^{-6}$	$\eta C$		$h$	$(Nu/RePr)_{th} \cdot 10^4$ , Eq. (18)	$(Nu/RePr)_{ex} \cdot 10^4$ (Vyshenskii dependence [1])	Relative discrepancy, %
0,05	1,3	0,5	3	0,133	22,53	25,29	11,0
0,02	1,3	0,5	3	0,133	13,6	15,93	15,0
0,03	1,3	0,5	3	0,133	15,95	20,7	23,0
0,04	1,3	0,5	3	0,133	19,47	21,96	11,3
0,06	1,3	0,5	3	0,133	24,89	30,14	17,4
0,05	0,65	0,5	3	0,133	26,85	25,29	6,2
0,05	0,37	0,5	3	0,133	30,65	25,29	21,2
0,05	0,9	0,5	3	0,133	24,68	25,29	2,5
0,05	1,3	0,3	3	0,133	21,42	27,12	21,1
0,05	1,3	0,4	3	0,133	22,25	26,91	17,3
0,05	1,3	0,6	3	0,133	22,25	24,09	7,6
0,05	1,3	0,7	3	0,133	21,42	22,93	6,6
0,05	1,3	0,5	2	0,133	27,40	31,21	12,2
0,05	1,3	0,5	2,5	0,133	25,9	27,88	7,1
0,05	1,3	0,5	3,5	0,133	19,71	23,57	16,4
0,05	1,3	0,5	4,0	0,133	17,49	22,01	20,5
0,05	1,3	0,5	3	0,07	22,71	22,66	0,2
0,05	1,3	0,5	3	0,08	22,46	26,58	15,5
0,05	1,3	0,5	3	0,1	22,77	26,42	13,9
0,05	1,3	0,5	3	0,15	22,58	26,10	13,5

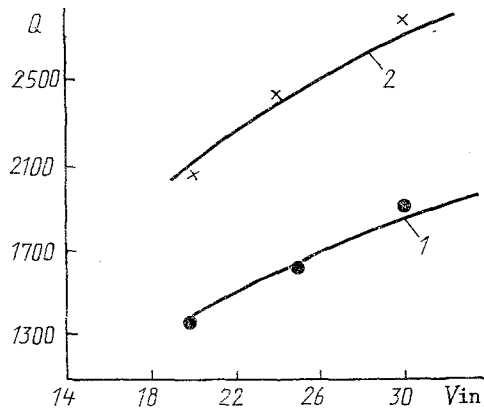


Fig. 2. Dependence of theoretical - from Eq. (22) - and experimental heat flux from the cylindrical surface of the cyclone chamber on the gas velocity at the cyclone input with  $T_{in} = 423$  (1) and  $473$  K (2); points) experiment; curves) calculation.  $Q$ , W;  $V_{in}$ , m/sec.

designs [1-4]. The results are:  $C_0 = 0.2$ ;  $m = 0.1$ ;  $K_1/K = 10$ , i.e., close to the analogous results for a plate ( $C_0 = 0.21$ ;  $m = 0.125$ ;  $K_1/K = 11$ ). The discrepancy between Eq. (18) and the formulas proposed in [2, 3] is less than 5%; the discrepancy with [4] is 10-15% (Fig. 1). Note that using the Vyshenskii formula [1] to calculate the heat-transfer coefficient from a dust-free gas to the cyclone wall, which also allows the influence of the structural dimensions of the chamber on  $\alpha$  to be taken into account, gives a greater (up to 30%) discrepancy in Nu (Fig. 1).

Table 1 gives the results of calculations by Eq. (18) and the Vyshenskii empirical dependence [1]. The limits of variation in the geometric characteristics of the cyclones are specified by the region of application of Eqs. (19) and (20) and the Vyshenskii formula. The convergence may be regarded as satisfactory; the discrepancy is no more than 15-20%.

Regrettably, only dependences of the form  $Nu = f(Re)$  are given in these works. In [8], experimental values of the heat fluxes to the cyclone wall from pure and dusty gas were given.

As the gas moves along the lateral surface, the temperature at the boundary of the flow core  $T_b$  changes. On the basis of the assumption that the local heat-transfer coefficient is constant over the length of the cyclone, as is its surface temperature  $T_{wa}$ , the following expression may be written

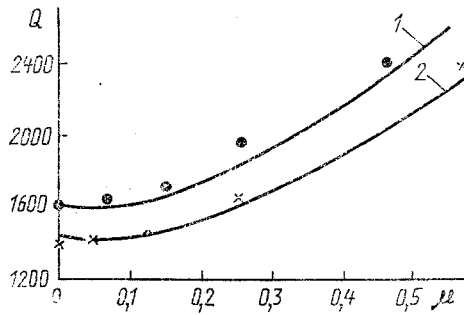


Fig. 3. Dependence of theoretical - from Eq. (22) - and experimental [8] heat flux from cylindrical surface of cyclone chamber on the gravimetric concentration of solid phase: 1)  $V_{in} = 25$ ; 2) 20 m/sec.

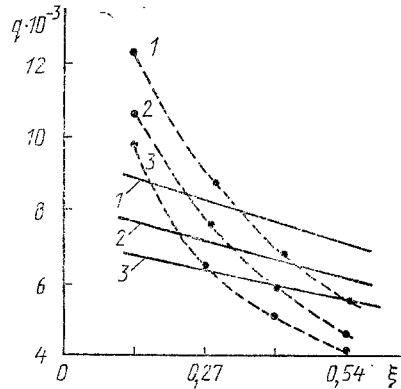


Fig. 4. Distribution of experimental [8] and theoretical - from Eq. (21) - specific heat fluxes over the cyclone length with  $V_{in} = 30$  (1), 25 (2), and 20 (3) m/sec; continuous curves) calculation; dashed curves) experiment.

$$\frac{dT_b}{dx} = \frac{\alpha 2\pi R_{cy}(T_b - T_{wa})}{G_g C_{ef}}$$

Taking into account that  $T = T_{in}$  when  $x = 0$ , it follows that

$$T_b = T_{wa} + (T_{in} - T_{wa}) \exp - \frac{2\pi R_{cy} \alpha x}{G_g C_{ef}},$$

and the current heat-flux density is

$$q_x = \alpha(T_b - T_{wa}) = \alpha(T_{in} - T_{wa}) \left[ 1 - \exp - \frac{2\pi R_{cy} \alpha x}{G_g C_{ef}} \right]. \quad (21)$$

The total heat flux from the cyclone surface is

$$Q = 2\pi R_{cy} \int_0^{L_{cy}} q_x dx = G_g C_{ef} (T_{in} - T_{wa}) \left[ 1 - \exp - \frac{2\pi R_{cy} \alpha L_{cy}}{G_g C_{ef}} \right]. \quad (22)$$

The total heat fluxes for the cylindrical part of the cyclone from Eq. (22) and the experimental data of [8] for pure gas with various input temperatures of the flux in the cyclone are compared in Fig. 2: 1)  $T_{in} = 150^\circ\text{C}$ ; 2)  $200^\circ\text{C}$ . It is evident that the discrepancy between the theoretical and experimental data is slight.

On the basis of a comparison of the theoretical and experimental heat fluxes, the empirical coefficient  $A$  in Eq. (18) is determined for dusty gas. It is evident from Fig. 3 that when  $A = 2$  the theoretical curves are in fair agreement with the experimental data.

Data on the distribution of specific heat fluxes  $q$  over the cyclone height were also obtained in [8]. The theoretical - from Eq. (21) - and experimental distributions of  $q$  are compared in Fig. 4. It is evident that the greatest difference in specific heat fluxes is seen in the upper part of the cyclone. The experimental values of  $q$  are considerably higher than the theoretical values. This is explained in that the model does not take into account that the flux is not yet stabilized in the upper end of the cyclone and hence the heat-transfer intensity in this region is higher than in the stabilized-flow zone. In addition, in deriving Eq. (18), it was assumed that the gas flow rate remains constant as it moves along the cyclone wall. In fact, there is discharge of the gas in the central zone (in the discharge tube).

Overall, it may be concluded, on the basis of these investigations, that Eq. (18) may be used to calculate the mean heat-transfer coefficient from swirling flow to the cyclone wall. In other words, the two-layer model of turbulent flow around a wall may be used to obtain an expression which permits the calculation of the heat-transfer coefficient from the gas to the cyclone wall from the known aerodynamic characteristics of the chamber, without the need for direct thermal measurements.

#### NOTATION

$C_g$ ,  $C_s$ , specific heat of gas and solid phases, J/kg·K;  $\mu$ , ratio of mass flow rates of solid and gas phase;  $a$ ,  $a_T$ , molecular and turbulent thermal diffusivity,  $m^2/sec$ ;  $\rho$ , density of gas,  $kg/m^3$ ;  $T$ , gas temperature, K;  $\nu$ ,  $\nu_T$ , molecular and turbulent kinematic viscosity of gas,  $m^2/sec$ ;  $V$ , tangential component of gas velocity,  $m/sec$ ;  $r$ , current radius,  $m$ ;  $R_{cy}$ , cyclone radius,  $m$ ;  $\ell$ , length of mixing path,  $m$ ;  $H$ , height of cyclone input slits,  $m$ ;  $\Sigma f$ , total area of input slits in cyclone,  $m^2$ ;  $R_C$ , radius of constriction in cyclone,  $m$ ;  $G_g$ , mass flow rate of gas,  $kg/sec$ ;  $x$ , current length of cyclone,  $m$ ;  $T_{in}$ , gas temperature at cyclone input, K.

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